COMPETITION COACH TOOLKIT

This is a collection of lists, formulas and terms that Mathletes frequently use to solve problems like those found in this handbook. There are many others we could have included, but we hope you find this collection useful.

Fraction	Decimal	Percent
1/2	0.5	50
1/3	$0.\overline{3}$	$33.\overline{3}$
1/4	0.25	25
1/5	0.2	20
1/6	$0.1\overline{6}$	$16.\overline{6}$
1/8	0.125	12.5
1/9	$0.\overline{1}$	$11.\overline{1}$
1/10	0.1	10
1/11	$0.\overline{09}$	$9.\overline{09}$
1/12	$0.08\overline{3}$	$8.\overline{3}$

Common Arithmetic Series

$$1 + 2 + 3 + 4 + \cdots + n = \frac{n(n+1)}{2}$$

$$1 + 3 + 5 + 7 + \cdots + (2n - 1) = n^2$$

$$2 + 4 + 6 + 8 + \cdots + 2n = n^2 + n$$

Prime Numbers

2	43
3	47
5	53
7	59
11	61
13	67
17	71
19	73
23	79
29	83
31	89
37	97
41	

Combinations & Permutations

$${}_nC_r = \frac{n!}{r!(n-r)!} \qquad {}_nP_r = \frac{n!}{(n-r)!}$$

n	n^2	n^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728
13	169	2197
14	196	2744
15	225	3375

Divisibility Rules

- 2: units digit is 0, 2, 4, 6 or 8
- 3: sum of digits is divisible by 3
- 4: two-digit number formed by tens and units digits is divisible by 4
- 5: units digit is 0 or 5
- 6: number is divisible by both 2 and 3
- 8: three-digit number formed by hundreds, tens and units digits is divisible by 8
- 9: sum of digits is divisible by 9
- 10: units digit is 0

Geometric Mean

$$\frac{a}{x} = \frac{x}{b} \quad \text{and} \quad x = \sqrt{ab}$$

Equation of a Line

Standard Form

$$Ax + By = C$$

Slope-Intercept Form

$$y = mx + b$$

m = slope b = y-intercept

Point-Slope Form

$$y - y_1 = m(x - x_1)$$

m = slope (x_1, y_1) = point on the line

Distance Traveled

Distance = Rate × Time

Quadratic Formula

For $ax^2 + bx + c = 0$, where $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagorean Triples

(3, 4, 5)	(5, 12, 13)	(7, 24, 25)
(8, 15, 17)	(9, 40, 41)	(12, 35, 37)

Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Sum and Difference of Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Circles

Circumference	Area
$2 \times \pi \times r = \pi \times d$	$\pi \times r^2$

For radius r

Arc Length	Sector Area
$\frac{x}{360} \times 2 \times \pi \times r$	$\frac{x}{360} \times \pi \times r^2$

For central angle of x degrees

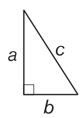
Given $A(x_1, y_1)$ and $B(x_2, y_2)$

Distance from A to B = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint of $\overline{AB}$ = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

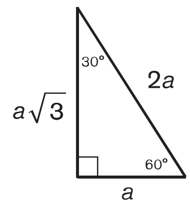
Slope of $\overline{AB}$ = $\frac{y_2 - y_1}{x_2 - x_1}$

Pythagorean Theorem

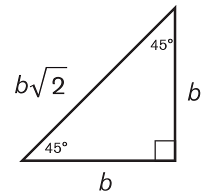


$a^2 + b^2 = c^2$

Special Right Triangles



30-60-90 Right Triangle



45-45-90 Right Triangle

Area of Polygons

Square	side length s	s^2
Rectangle	length l, width w	$l \times w$
Parallelogram	base b, height h	$b \times h$
Trapezoid	bases b_1, b_2, height h	$\frac{1}{2}(b_1 + b_2) \times h$
Rhombus	diagonals d_1, d_2	$\frac{1}{2} \times d_1 \times d_2$
Triangle	base b, height h	$\frac{1}{2} \times b \times h$
Triangle	semi-perimeter s, side lengths a, b, c	$\sqrt{s(s-a)(s-b)(s-c)}$
Equilateral Triangle	side length s	$\frac{s^2\sqrt{3}}{4}$

Polygon Angles (*n* sides)

Sum of the interior angle measures:

$$180 \times (n - 2)$$

Central angle measure of a regular polygon:

$$\frac{360}{n}$$

Interior angle measure of a regular polygon:

$$\frac{180 \times (n - 2)}{n} \quad \text{or} \quad 180 - \frac{360}{n}$$

Solid	Dimensions	Surface Area	Volume
Cube	side length s	$6 \times s^2$	s^3
Rectangular Prism	length l, width w, height h	$2 \times (l \times w + w \times h + l \times h)$	$l \times w \times h$
Circular Cylinder	base radius r, height h	$2 \times \pi \times r \times h + 2 \times \pi \times r^2$	$\pi \times r^2 \times h$
Circular Cone	base radius r, height h	$\pi \times r^2 + \pi \times r \times \sqrt{r^2 + h^2}$	$\frac{1}{3} \times \pi \times r^2 \times h$
Sphere	radius r	$4 \times \pi \times r^2$	$\frac{4}{3} \times \pi \times r^3$
Pyramid	base area B, height h		$\frac{1}{3} \times B \times h$